PHYS*4150 Review

Solid State Physics

Course outline

- Free electron model
  - Drude
  - Sommerfeld
- Crystal lattices
  - Direct lattice
  - Reciprocal lattice
- Diffraction
- Semi-classical theory of metals
  - Electrons in crystal lattice
  - Bloch's theorem
  - Nearly-free electron model
- Dynamic lattice (phonons)

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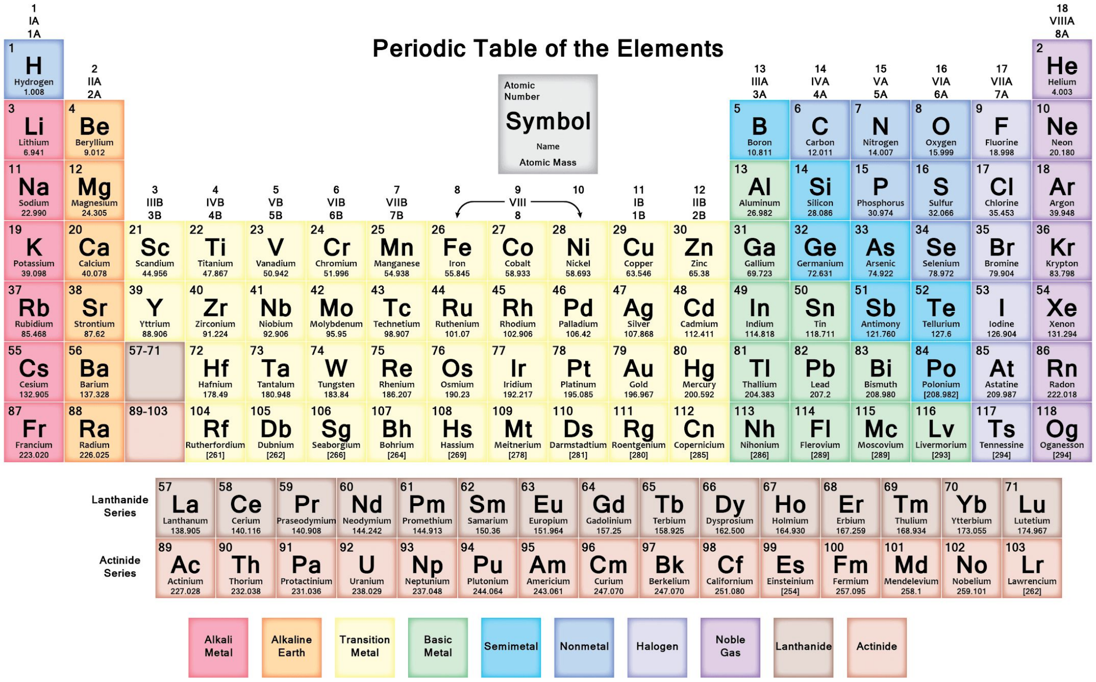
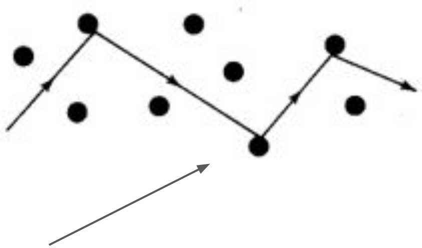
Metals



Free electron model

Drude Model

Classical treatment of electrons in a metal

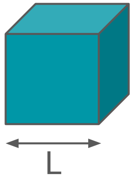


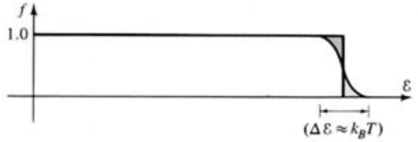
Phenomenological scattering time τ

Sommerfeld Model

Application of Quantum mechanics and Fermi-Dirac statistics to Drude model

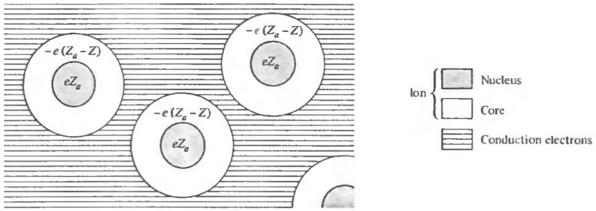
$$\psi(x,y,z+L)=\psi(x,y,z)$$
$$\psi(x,y+L,z)=\psi(x,y,z)$$
$$\psi(x+L,y,z)=\psi(x,y,z)$$





Drude model

Classical model of free electron gas



Independent electron approx.

Free electron approx.

Classical laws of motion

DC conductivity: resistivity, current density

AC conductivity: reflectivity, transparency, plasma frequency, skin depth

Hall effect: Hall coefficient

Thermal conductivity: thermal conductivity coefficient, thermopower (Seebeck effect)

FAILURES of model

Sommerfeld model

Quantum mechanics: free electron with periodic boundary conditions

Pauli exclusion principle: Fermi-Dirac statistics, chemical potential

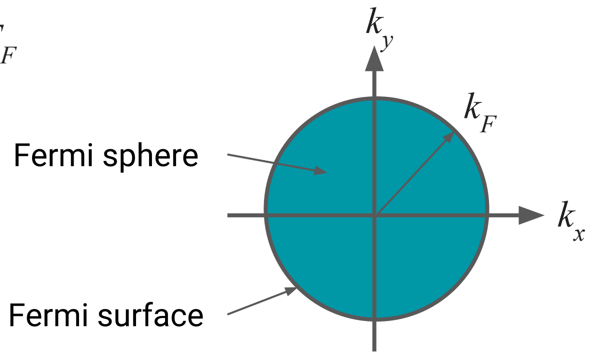
Plane-wave solutions: introduction to k-space

Density of states

Corrections to c_V and v_0

Fermi terminology

E_F, k_F, v_F, T_F



$$\sum_{\mathbf{k}} F(\mathbf{k}) = \frac{V}{8\pi^3} \sum_{\mathbf{k}} F(\mathbf{k})\Delta\mathbf{k} \rightarrow \frac{V}{8\pi^3} \int F(\mathbf{k}) d\mathbf{k}$$

Crystal lattices

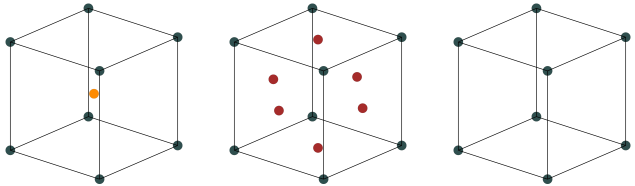
Bravais lattices: primitive vectors, lattice vectors → $R(a_1,a_2,a_3)$, unit cell, Wigner-Seitz cell

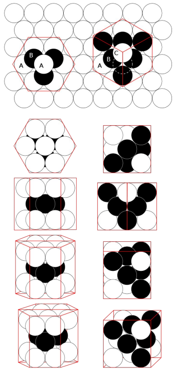
Cubic lattices: Simple cubic, FCC, BCC

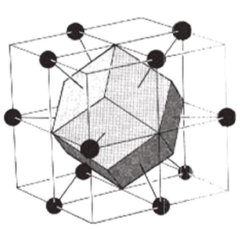
Lattice with a basis

Close-packed structures: HCP, FCC

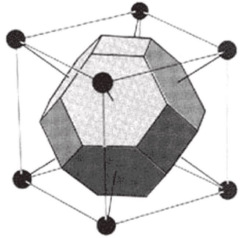
$$\bar{R} = n_1\bar{a}_1 + n_2\bar{a}_2 + n_3\bar{a}_3$$







Face Centered Cubic Wigner-Seitz Cell



Body Centered Cubic Wigner-Seitz Cell

Reciprocal lattice

Fourier transform of direct lattice

More k-space

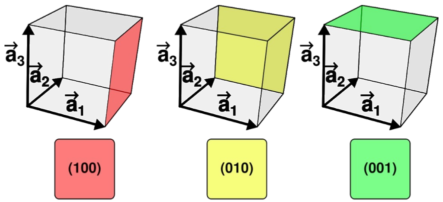
Bravais lattice: primitive vectors, reciprocal lattice vectors → $K(b_1, b_2, b_3)$

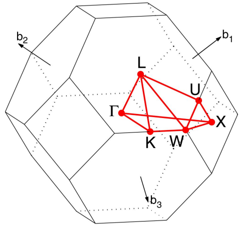
Miller planes

Brillouin Zones

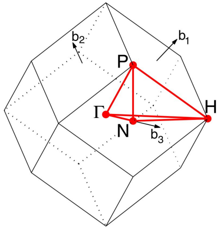
$$e^{i\bar{K}\cdot\bar{R}} = 1$$

$$\bar{K} = n_1\bar{b}_1 + n_2\bar{b}_2 + n_3\bar{b}_3$$





FCC path: Γ-X-W-K-Γ-L-U-W-L-K|U-X



BCC path: Γ-H-N-Γ-P-H|P-N

Diffraction

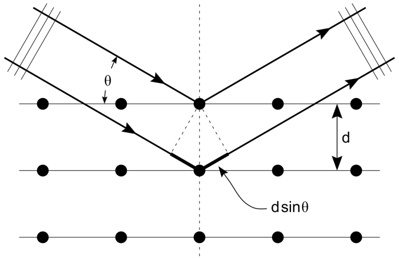
Study/measurement of crystal lattices

X-ray diffraction: direct measurement of reciprocal lattice

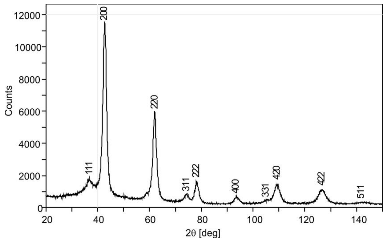
Von Laue diffraction

Powder diffraction

Structure factor: lattice with a basis, intensity of diffracted peaks



$$\bar{K} = \bar{k} - \bar{k}'$$

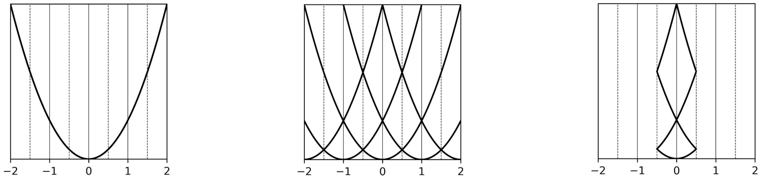


$$S_{\bar{K}} = \sum_j e^{i\bar{K}\cdot\bar{d}_j} \qquad I_{\bar{K}} \propto |S_{\bar{K}}|^2$$

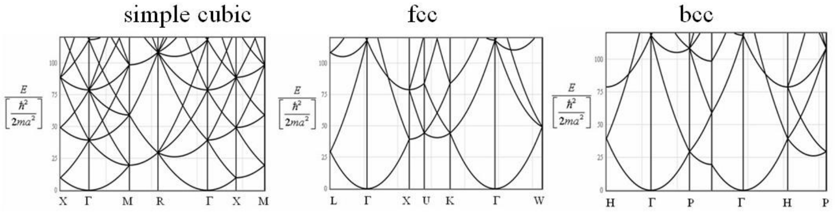
Electrons in a crystal lattice

Integration of crystal lattices with free electron model: empty lattice approximation, 2N electron states per B.Z., reduction to 1st B.Z. → bands

Bloch's Theorem: electrons in a periodic potential



Extended Repeated Reduced



Plane waves (FEM)

$$\psi_{n,k}(\bar{r}) = e^{i\bar{k}\cdot\bar{r}} u_{n,k}(\bar{r})$$

Lattice modulation

$$\psi_{n,k}(\bar{r}+\bar{R}) = e^{i\bar{k}\cdot\bar{R}}\psi_{n,k}(\bar{r})$$

$$u_{n,k}(\bar{r}+\bar{R}) = u_{n,k}(\bar{r})$$

Nearly free electron

Weak periodic potential: treat with perturbation theory

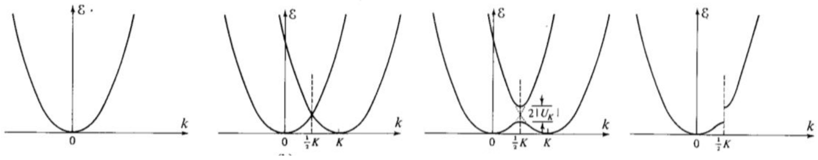
Band gap = $2|V_G|$

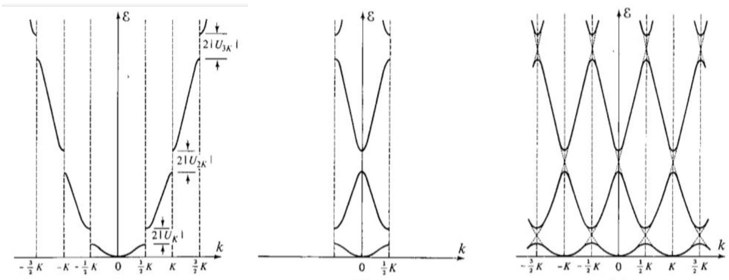
Material types

Insulator requires even # of electrons

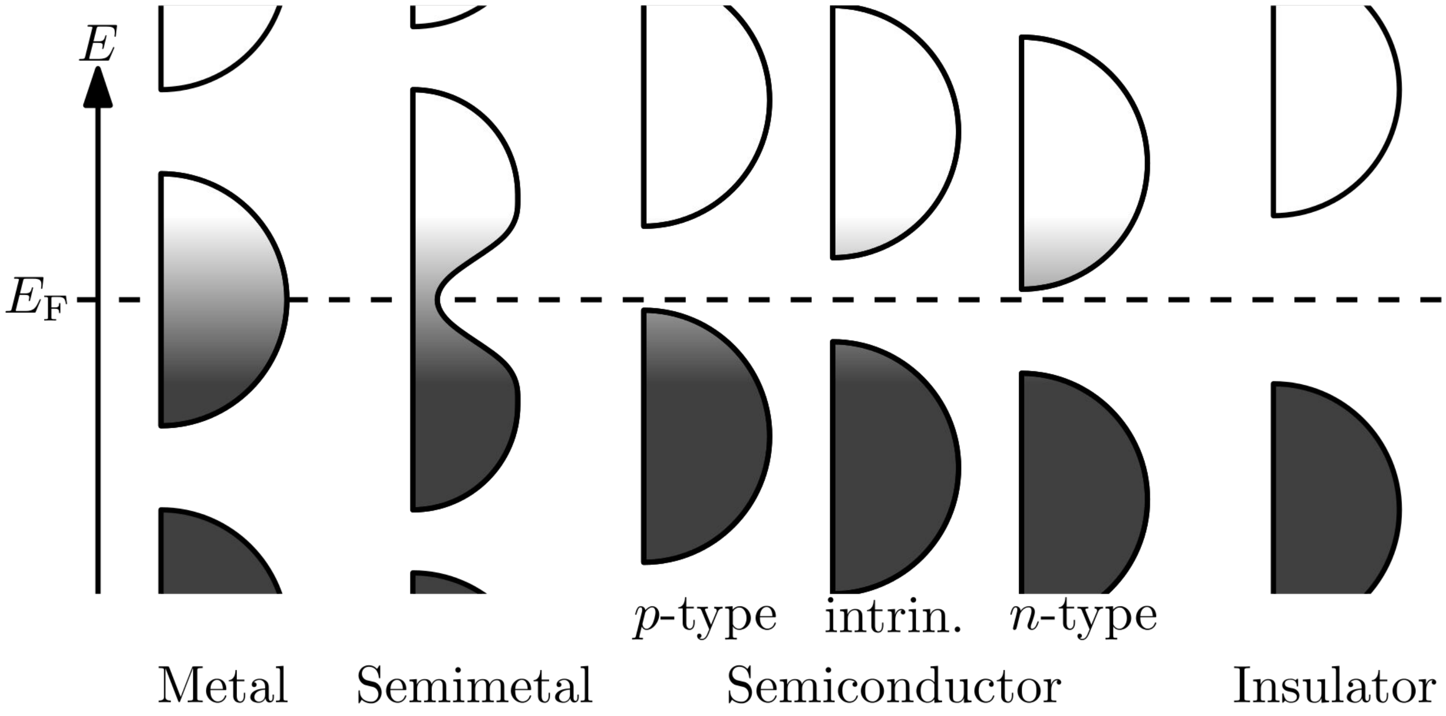
Filled bands are inert

$$V(x) = \sum_{\bar{G}} V_{\bar{G}} e^{i\bar{G}\cdot\bar{r}}$$



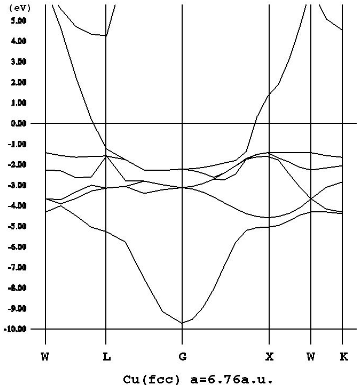


Materials

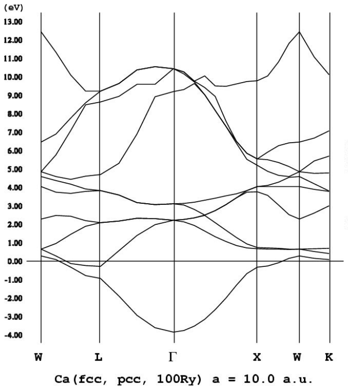


E
E_F
p-type
intrin.
n-type
Metal
Semimetal
Semiconductor
Insulator

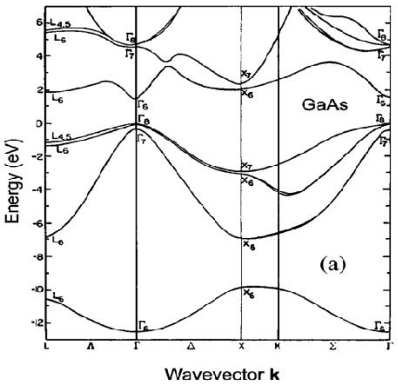
Band structure



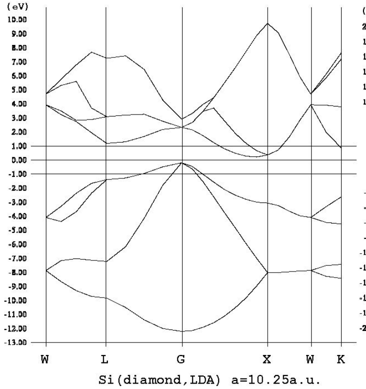
Metal



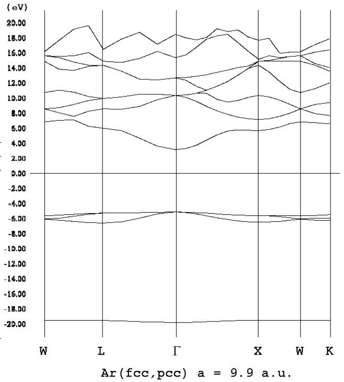
Semi-metal



Semiconductor
(direct)



Semiconductor
(indirect)



Insulator

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Dynamic lattice

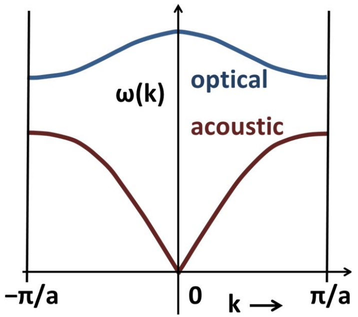
Harmonic lattice approximation

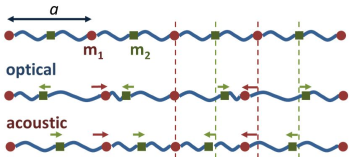
$$\varphi^0(l,l') = \left.\frac{\partial^2 W}{\partial u_l \partial u_{l'}}\right|_0$$

Dynamical matrix: dispersion curves, 1D monatomic, diatomic lattice, optic and acoustic modes, generalize to 3D

$$\hat{D}^{\mu,\mu'}_{\alpha,\alpha'}(\bar{k}) = \frac{1}{\sqrt{M_\alpha M_{\alpha'}}} \sum_m \varphi^{\mu,\mu'}_{\alpha,\alpha'}(m) e^{i\bar{k}\cdot \bar{R}^0_m}$$

$3*N_b*N$ normal modes: N modes per Brillouin zone per branch





Phonons

$$Energy = \hbar\omega_{n,k}$$

$$n_{s,k} = \frac{1}{e^{\hbar\omega_{s,k}/k_B T} - 1}$$

Occupancy: # of phonons in state s,k

Final Exam:
Wed. April 17, 8:30-10:30am, MacN318